Differences between the original and current general relativity

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Introduction

- The original theory of general relativity (OGR) was published by A. Einstein in 1915.
- About two decades later, the theory was applied to model the relativistic compact objects (RCOs), like neutron stars.
- In this application, some additional, restrictive, assumptions were adopted. In consequence, the original theory was modified in several aspects.
- The general relativity (GR) that is today the main-stream theory of gravity is, thus, different from the original, more general theory.
- In this talk, the essential differences between the OGR and GR will be pointed out.

RCO in the original theory

- A spherically symmetric RCO can be model solving the field equations in the form

$$\kappa T_1^1 = -\kappa P = -\mathrm{e}^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2}, \tag{1}$$

$$\kappa T_2^2 = -\kappa P = -\mathrm{e}^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right), \tag{2}$$

$$\kappa T_3^3 = -\kappa P = -\mathrm{e}^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right), \tag{3}$$

$$\kappa T_4^4 = \kappa\tilde{\rho} = \mathrm{e}^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}. \tag{4}$$

- These four equations contain four unknown quantities, P, $\tilde{\rho}$, λ, and ν.
- Notice, however, that the components T_1^1, T_2^2, and T_3^3 of the stress-energy tensor for a perfect fluid are identical, therefore Eqs.(2) and (3) are the same and, thus, we have only three equations, in fact, but four unknown quantities remain.

Categories of the fourth equation

- The fourth equation must be supplied from outside of OGR or GR to complete the system of equations.
- Two categories of the fourth equation:

  A. Realistic case $=>$ realistic equation of state (EoS) like that for cool, degenerated, Fermi-Dirac, neutron gas, or polytrope, EoS for radiation, etc.

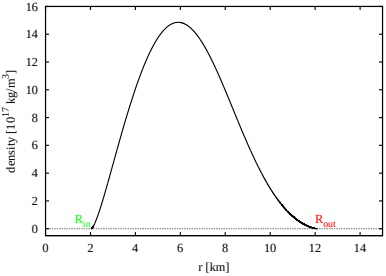
  B. Mathematical experiment with the field equations $=>$ whatever equation, also an equation originated in human fantasy, etc.

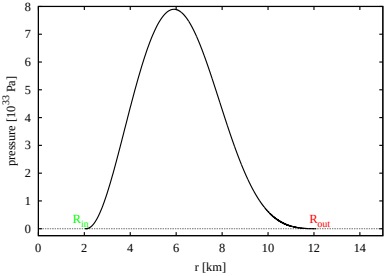
- The solution of system of equations with the 4th equation of category B should never be used in an argument in real applications.

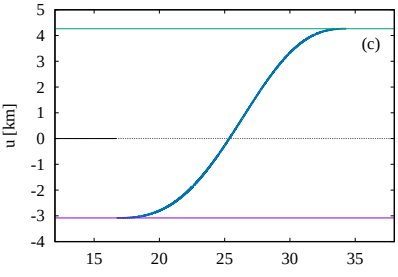
OGR - a typical model of RCO

Behavior of density, pressure, and auxiliary metric function $u = (r/2)(1 + 1/g_{rr})$:

Unit of u... meter;
unit of uc^2/G... kilogram;
$u(r \leq R_{in}) \equiv u_{in} = const.$;
$u(R_{out} \geq) \equiv u_{out} = const.$







Constants $u_c = u_{out}$ (or $u_c = u_{in}$) are present in the outer Schwarzschild solution of field equations, specifically

$$g_{rr} = \left(1 - \frac{2u_c}{r}\right)^{-1}, \qquad g_{tt} = K_\nu \left(1 - \frac{2u_c}{r}\right)$$

Inner surface

- In OGR, the inner surface of RCO must always occur because of following reason.
- The formula for the acceleration derived from the equation of geodesic is (Straumann, N.: 2013, General Relativity, Springer, p. 59)

$$\frac{d^2x^\alpha}{dt^2} = \left(\Gamma^4_{\beta\gamma}\frac{dx^\alpha}{dt} - \Gamma^\alpha_{\beta\gamma}\right)\frac{dx^\beta}{dt}\frac{dx^\gamma}{dt}, \tag{5}$$

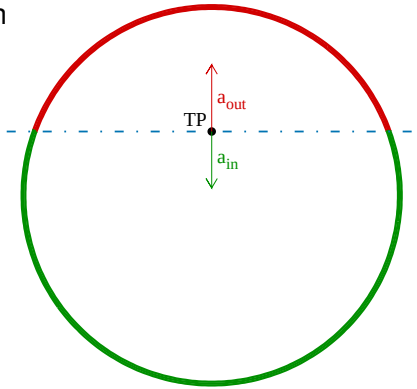
  where $\Gamma^\gamma_{\alpha\beta}$ are the Christoffel symbols.
- If we consider a test particle (TP) being in rest ($dr/dt = d\vartheta/dt = d\varphi/dt = 0$; $dt/dt = 1$) in respect to a point-like massive particle (both are in vacuum), then the formula for the radial component can be simplified to

$$\frac{d^2x^1}{dt^2} = a = -c^2\Gamma^1_{44} = \frac{c^2}{2}\frac{1}{g_{rr}}\frac{dg_{tt}}{dr} = -\left(1 - \frac{R_g}{r}\right)\frac{GM_N}{r^2}. \tag{6}$$

  R_g is the gravitational radius and M_N Newtonian mass of the massive particle.

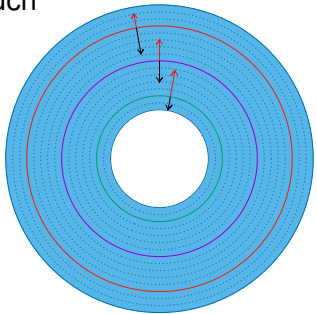
Inner surface

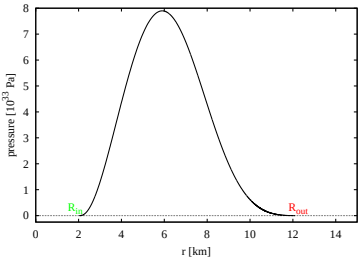
- Let us consider a thin spherical shell with a weak gravity.
- If we take into account only the first term in the acceleration formula, i.e. acceleration implied by the Newton's law $a = -GM_N/r^2$, then the size of net acceleration, $|a_{out}|$, of the shell's red globular canopy is the same as the size of net acceleration, $|a_{in}|$, of the green canopy and TP does not accelerate.
- However, if also the second term, $+GM_N R_g/r^3$, is taken into account, then the gravity of the red canopy is dominant and the TP is accelerated away from the center of the shell.



Zero-gravity distace

- In a stable gaseous object, the gravity is balanced by the gradient of pressure.
- In a region near the outer surface, the downward oriented net gravity of lower layers dominates and is balanced by the outward oriented gradient.
- In a region near inner surface, the outward oriented net gravity of outer layers is dominant, on contrary, and is balanced by the inward oriented gradient (see the opposite slope of the behavior of pressure in the right-side plot).
- Inside the object, there is such a distance, in which the net gravity of inner layers is just equal to the oppositely oriented net gravity of outer layers; the total gravity is zero, here.
- We refer to this distance as the **zero-gravity distance**.





Concept of mass

- The main difference between the OGR and GR was caused by the demand (postulate), in the GR, that only “regular solutions” of field equations can be used to describe a real object.
- This means, in consequence, that the relativistic mass of a RCO, M, is always identified with the mass, M_N, in the Newtonian physics.
- In both, the OGR and GR, mass is related to the energy, E, by the well-known Einstein’s formula $E = Mc^2$; symbol c stands for the speed of light.
- The energy is the integral of energy density, $\tilde{\rho}$, through the 3D volume of the object. In the case of spherical symmetry, the integrations through two transversal components give $4\pi r^2$, therefore the relativistic mass can be given as

$$M = \frac{1}{c^2} \int_{R_{in}}^{R_{out}} 4\pi r^2 \tilde{\rho}\, dr. \tag{7}$$

Concept of mass

- Field equations imply

$$\frac{du}{dr} = \frac{1}{2}\kappa\tilde{\rho}r^2, \tag{8}$$

from which $\tilde{\rho}r^2 = (2/\kappa)du/dr$ and

$$M = \frac{4\pi}{c^2}\int_{R_{in}}^{R_{out}} \tilde{\rho}r^2\,dr = \frac{8\pi}{c^2\kappa}\int_{R_{in}}^{R_{out}} \frac{du}{dr}\,dr =$$
$$= \frac{c^2 u_{out}}{G} - \frac{c^2 u_{in}}{G} = M_{out} - M_{in} = M_{out} + |M_{in}|, \tag{9}$$

where $\kappa = 8\pi G/c^4$ is the Einstein gravitational constant. We also denoted $c^2 u_{in}/G = M_{in}$ and $c^2 u_{out}/G = M_{out}$.

- We note that there is always valid $u_{out} > 0$ and $u_{in} < 0$, hence $M_{out} > 0$ and $M_{in} < 0$, respectively. Relativistic mass, M, is then always $M > 0$.
- Parameter M_{out} is measured above the outer surface of the object as its 'mass'. It can be identified with the Newtonian mass, M_N.

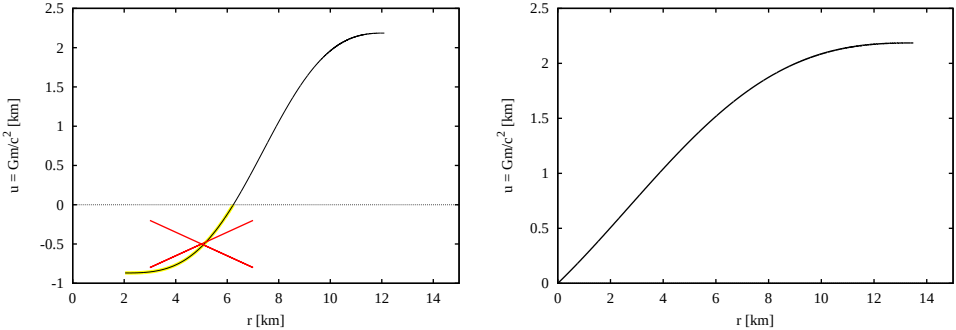
Mass in the GR

- In the GR, there is defined the *mass within the sphere of radius r*,

$$m_r = \frac{1}{c^2}\int_0^r 4\pi r^2 \tilde{\rho}\, dr = \frac{8\pi}{c^2\kappa}\int_0^r \frac{d[Gm(r)/c^2]}{dr}\, dr = \frac{8\pi}{c^2\kappa}\frac{G}{c^2}\int_0^r \frac{dm(r)}{dr}\, dr$$

  (where $u(r) = Gm(r)/c^2$).
- Regular solution is that with the inner border of integration equal to zero, $R_{in} = 0$.
- It is reasonable to assume that the mass is always non-negative, therefore the part of function u (or m) below the x-axis must be eliminated when $c^2 u/G = m$ is represented as a *mass*.
- This is achieved postulating zero in the lower border of the integral.
- If $R_{in} \to 0$, then $u_{in} \to 0$ (then $M_{in} \to 0$) and $M = M_{out} \equiv M_N$.

Acceptable solutions in the GR

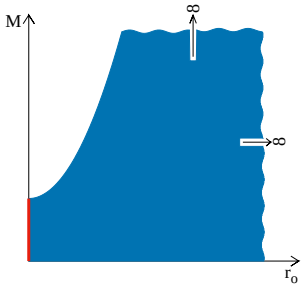


Such a solution of field equations is searched for that the relativistic term in the formula for acceleration is negligible (and ignored in a small region around the object's center):

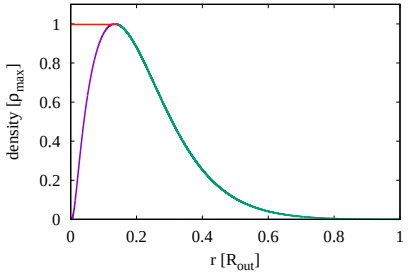
$$a = -\frac{GM_N}{r^2}\left(1 - \frac{R_g}{r}\right) \quad \rightarrow \quad a = -\frac{GM_N}{r^2}$$

Extent of OGR and GR solutions

However, the requirement $R_{in} = 0$ means that an essential part (corresponding to blue area) of the solutions of field equations is forbidden, when a realistic model of RCO is created.



Within the GR, a model of RCO is created via a numerical integration from a small distance r_{start} to R_{out}; the values of all quantities in the interval $(0, r_{start})$ are assumed to be the same as in r_{start}.

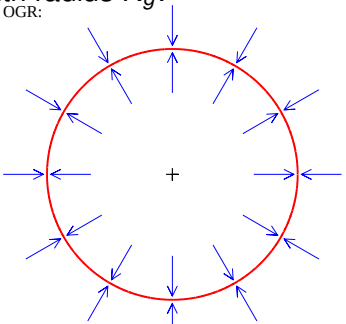


Using the field equations and realistic equation of state, Chuck Norris constructed a model of neutron star in the form of rigorously full sphere.

Orientation of acceleration

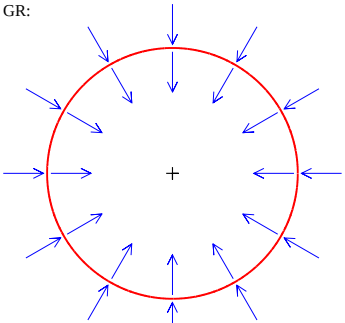
Sphere with radius R_g:

OGR:



$$a_{OGR} = -\frac{GM_N}{r^2}\left(1 - \frac{R_g}{r}\right)$$

GR:



$$a_{GR} = -\frac{GM_N}{r^2}$$

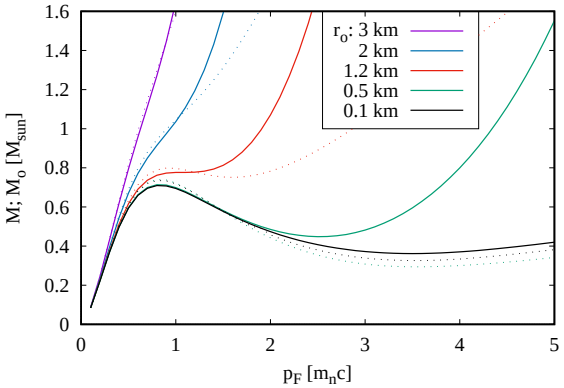
- In the OGR: a TP is accelerated toward the sphere of radius R_g.
- In the GR: a TP is accelerated toward the center.

Gravitational collapse?

- In the GR, a RCO can collapse onto the center; its outer radius can decrease below its event horizon; the formation of black hole is possible.
- In the OGR, the outer radius could approach the event horizon (i.e. the outer radius to the gravitational radius) if the inner radius also approaches the event horizon and the thickness of the object, in form of shell, approaches zero.

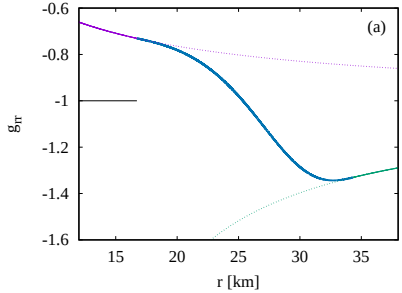
- However, deLyra (deLyra J.L.: 2024, Gen. Rel. & Grav. 56, 136) demonstrated that the energy increases above all limits when the thickness approaches zero; the collapse onto the event horizon is energetically forbidden in the OGR.

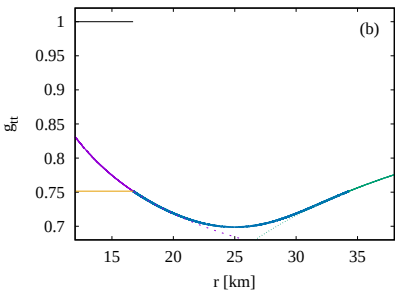
Upper mass limit

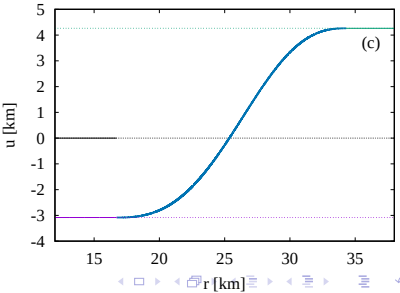


- In the GR, the mass of stable object is limited; Oppenheimer-Volkoff limit (see the local maximum of black curve in the plot).
- In the OGR, with a various possible zero-gravity distance, the mass of stable object is not constrained (see the blue and purple curves in the plot).

Continuity of metrics







- OGR: Metrics is continuous in $r \in (0, \infty)$ – purple-thick blue-green solid curves.
- GR: The postulate of Minkowski metrics in the interior of a shell implies a discontinuity of metrics at the inner surface of the shell – black (orange) solid-line segments.

Summary of differences

OGR:

- Relativistic mass approaches Newtonian mass only in the limit of weak field.
- Mass of super-massive objects is typically several orders of magnitude larger than the Newtonian mass.
- A test particle is attracted toward the sphere with the center identical to the center of acting particle and with radius R_g.
- Gravitational acceleration is oriented outward in the innermost region of compact object.

GR:

Relativistic mass is postulated to be identical with the mass in the Newtonian physics.

Mass of whatever object is identical with its Newtonian mass.

A test particle is attracted toward the acting particle in a whatever distance from the acting particle.

Gravitational acceleration is always oriented toward the center of the object.

Summary of differences

OGR:

- No upper-mass limit of stable object exists.
- Collapse of object onto a sphere with the outer radius equal or smaller than gravitational radius is energetically forbidden.
- The metrics is continuous from zero to infinity.
- There is a Big-Bang type, abstract singularity in $r = 0$.

GR:

There is the Oppenheimer-Volkoff upper-mass limit.

Eternal collapse of object below its event horizon is possible; an existence of black hole is possible.

There is a discontinuity of metrics at inner surface of shell.

There is a true, naked singularity in $r = R_g$.

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